The Calibration and Performance of a Microstrip Six-Port Reflectometer

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Abstract —**The calibration and performance of a microstrip six-port reflectometer consisting of only one six-port coupler is discussed. The positions of the centers of the impedance-locating circles are determined from the calibration constants, and their frequency behavior is illustrated. The results of measuring some terminations by this reflectometer and two HP network analyzers are compared within the frequency range from 0.5 GHz to 8 GHz. From this comparison the useful bandwidth of the reflectometer is found to be from 0.5 GHz to 5.5 GHz.**

I. Introduction

THE USE OF A coupled three-line system as a six-port reflectometer has been investigated in coaxial [1], waveguide [2], and microstrip [3]–[4] systems. The coaxial and waveguide versions were reported to have fairly narrow bandwidth. The theory of using a microstrip three-line system as a six-port reflectometer and a proposed "most suitable configuration" for this reflectometer were given [3]. This theory was developed [4] to allow better prediction and control of the performance of the six-port coupler formed from the coupled three-line system. The measured performance of this coupler [4] was found to be quite satisfactory from 2 GHz to about 6 GHz and is expected to be good below 2 GHz. The application of this coupler, at its center frequency, as a reflectometer was found to be quite good.

In this paper, the use of the coupler as reflectometer is investigated within the frequency range from 0.5 GHz to 8 GHz in an attempt to determine the useful bandwidth of the reflectometer. In Section II, both an approximate and an exact form of the equations of the reflectometer are presented. In Section III, the calibration of the reflectometer based on the approximate equations of the reflectometer is presented. The adequacy of this calibration procedure is checked by comparing its results with those of another one which is based on the exact form of the equations of the reflectometer. This comparison is carried out at the center, upper, and lower limits of the reflectometer's useful bandwidth.

According to the principle of operation of the six-port reflectometer [4], [5], the unknown impedance is determined from the intersection of three circles. These circles will be referred to as the impedance-locating circles. Their centers and radii are determined in terms of the calibration constants of the reflectometer. The frequency behavior of

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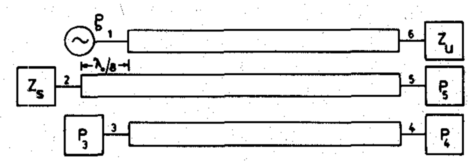

Fig. 1. The proposed configuration of the six-port reflectometer.

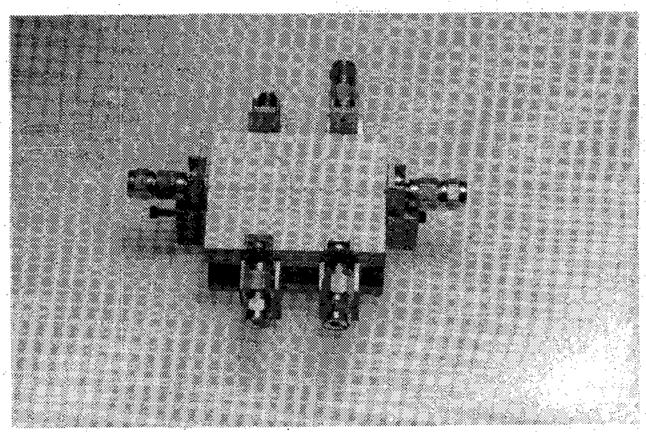

Fig. 2. Photograph of the experimental six-port reflectometer.

the positions of the centers is also illustrated in Section III.

The performance of the reflectometer is illustrated in Section IV where it is compared with that of the HP network analyzers 8505A and 8410B. In Section V, the useful bandwidth of the reflectometer is determined.

II. Equations of the Six-Port Reflectometer

The configuration of the microstrip six-port reflectometer to be calibrated is shown in Fig. 1. The unknown impedance Z_u is connected to the measuring port (port 6 in Fig. 1). Three power meters P_3, P_4, and P_5 are connected to ports 3, 4, and 5, respectively. A standard short-circuit Z_s is connected to port 2 and RF source to port 1. A photograph of the experimental six-port reflectometer is shown in Fig. 2.

The design goals of the present reflectometer were met quite satisfactorily within the frequency range from 2 GHz to about 6 GHz [4]. Consequently, approximate equations for the reflectometer can be written as [5]

$$P_3/P_0 = |A|^2|\Gamma_u + B|^2$$
$$P_4/P_0 = |C|^2|\Gamma_u + D|^2$$
$$P_5/P_0 = |E|^2|\Gamma_u + F|^2. \qquad (1)$$

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In (1), P_0 is the input power at port 1 and A, B, C, D, E, and F are constants that are generally complex. Then the nine real constants corresponding to $|A|$, $|C|$, $|E|$, B, D, and F are to be determined.

As described by Engen [5] the three expressions of (1) represent three circles in the Γ_u-plane. Since Γ_u should satisfy all of the three expressions of (1), the point of intersection of the corresponding three circles should determine Γ_u both in phase and magnitude. This latter fact is used to determine Γ_u after obtaining the calibration constants.

For an actual reflectometer, the expressions of (1) are not truly valid. The exact expressions relating Γ_u to P_3, P_4, P_5, and P_0 can be obtained from the general scattering matrix of the reflectometer. By using a procedure similar to that outlined in the appendix of [1] these expressions can be put in the following general form:

$$P_3/P_0 = |A|^2|\Gamma_u + B|^2/|1+G\Gamma_u|^2$$
$$P_4/P_0 = |C|^2|\Gamma_u + D|^2/|1+G\Gamma_u|^2$$
$$P_5/P_0 = |E|^2|\Gamma_u + F|^2/|1+G\Gamma_u|^2. \qquad (2)$$

It is clear from (2) that there are eleven real calibration constants corresponding to $|A|$, $|C|$, $|E|$, B, D, F, and G which are to be determined. Note that (2) reduces to (1) if $G = 0$. As will be shown in Section III, this is important when comparing results of the two calibration procedures based on (1) and (2).

It was shown [1] that an expression of the same form as those of (2) represent a circle in the Γ_u-plane. However, the center and radius of such a circle are dependent on Γ_u, which is not the case for the circles of (1). Here also, Γ_u is determined from the intersection of the three circles corresponding to the three expressions of (2).

III. Calibration of the Six-Port Reflectometer with Known Terminations

Several techniques have been reported for calibrating a six-port reflectometer (e.g., [6]–[9]). The present calibration technique is similar to that of Hoer [8] in that it basically uses four known terminations. The input power P_0 at port 1 in Fig. 1 was measured by inserting a calibrated 10-dB coupler between the RF source and port 1. This allows the calibration constants to be determined irrespective of the value of P_0. The calibration is then starting by connecting a termination of known reflection coefficient (both phase and magnitude) to port 6 in Fig. 1, and recording the corresponding readings of the three power meters P_3, P_4, and P_5 and the input power P_0. This provides us with one set of the expressions of (1) or three equations. Thus the connection of another two different and known terminations to port 6 and the recording of the corresponding powers will provide two more sets of the expressions of (1) or six equations. This yields nine equations that should theoretically suffice to determine the nine real constants of (1). However, due to measurement error, e.g., noise and errors in the detectors of the power meters and or the 10-dB coupler, it is preferable to use more known terminations. This results in redundant equations that can help in overcoming the resulting uncertainties in determining the calibration constants. Therefore, in the present calibration a total of seven terminations (a matched load and six positions of a sliding short-circuit) have been used. These terminations have been divided into two groups each containing the matched load and three positions of the sliding short-circuit, i.e., four terminations per group. The calibration constants are determined from each of the two groups in the way outlined in the Appendix. The use of two calibration groups allows cross checking of their results and consequently the exclusion of erroneous results. Thus their use is advantageous mainly for the primary calibration. For subsequent calibrations, because of the long-term stability of the reflectometer microstrip circuit, only one group should be quite sufficient.

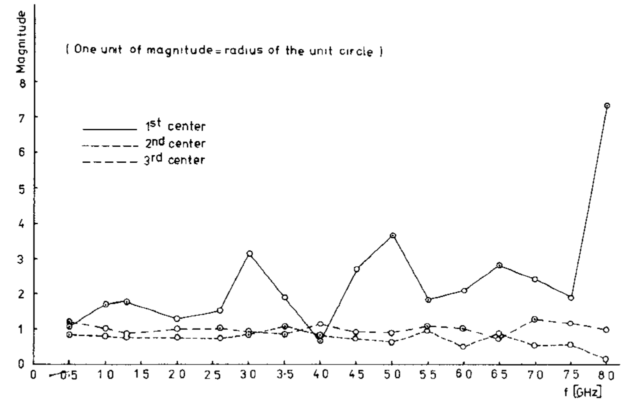

Fig. 3. Frequency dependence of the magnitudes of the vectors locating the three centers.

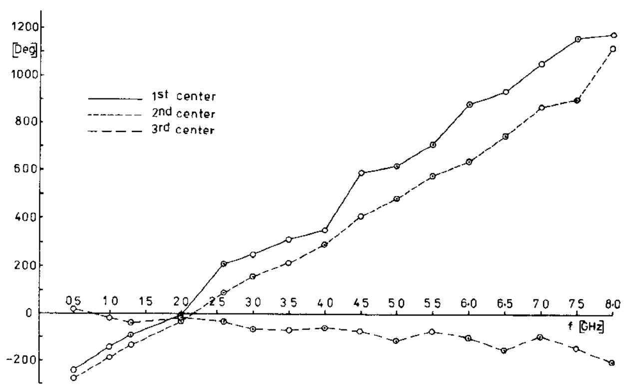

Fig. 4. Frequency dependence of the phases of the vectors locating the three centers.

By using the above procedure the calibration constants were determined at sixteen frequencies within the frequency range from 0.5 GHz to 8 GHz. Since the centers of the three impedance-locating circles corresponding to the expressions of (1) are located at $-B$, $-D$, and $-F$, their frequency behavior can be determined from the obtained values of the calibration constants. This behavior is illustrated in the graphs shown in Fig. 3 and Fig. 4 as the change in phase and magnitude of the vector pointing from the origin towards the considered center. The maximum frequency sensitivity of the positions of these centers, given

TABLE I
MAXIMUM FREQUENCY SENSITIVITY OF THE MAGNITUDE AND PHASE OF THE LOCATION FOR THE THREE CENTERS

Center	Maximum frequency sensitivity	
	Magnitude [MHz^{-1}]	Phase [Deg./MHz]
First	0.0041	0.48
Second	0.00093	0.24
Third	0.0011	0.12

TABLE II
COMPARISON OF THE RESULTS OF THE APPROXIMATE AND EXACT CALIBRATION PROCEDURES

Frequency [GHz]	Procedure	CALIBRATION CONSTANTS						
		|A|	|C|	|E|	B	D	F	G
0.5	approx.	0.0137	0.0175	0.058	0.45 -j0.95	-0.14 -j0.810	-1.12 -j0.36	0.0 +j0.0
	exact	0.0137	0.0195	0.067	0.16 -j1.05	-0.064 -j0.730	-0.96 -j0.32	-0.056 +j0.086
3.0	approx.	0.056	0.087	0.265	1.22 +j2.9	0.795 -j0.234	-0.33 +j0.80	0.0 +j0.0
	exact	0.059	0.088	0.262	1.45 +j2.61	0.797 -j0.200	-0.32 +j0.82	0.030 +j0.005
6.0	approx.	0.031	0.094	0.228	1.95 -j0.68	0.04 +j0.454	0.229 +j0.97	0.0 +j0.0
	exact	0.027	0.094	0.229	2.33 -j0.25	-0.11 +j0.467	0.168 +j0.99	-0.030 +j0.141

as the maximum slope between two successive points in the graphs of Fig. 3 and Fig. 4, is shown in Table I.

The adequacy of this approximate calibration procedure is checked by comparing its results with those of another procedure based on the exact expressions of (2).

It is clear from the expressions of (2), and the approximate calibration procedure, that a minimum of four known terminations, both in phase and magnitude, are necessary to determine the present calibration constants. These terminations will provide us with twelve equations. This system of equations was solved by using a minimization subroutine. This subroutine needs an initial guess for each of the eleven constants to be determined. It minimizes the sum of the squares of the residuals of the above twelve equations, put in the form $f(x)=0$, after substituting the initial guess values of the eleven constants. The values of the nine constants obtained from the approximate procedure, plus putting $G=0.0+j0.0$, were used as the initial values. This is justified since a small value of G is expected for the reflectometer circuit. Consequently, the nine real constants of (1) are expected to be near to the corresponding ones in (2). This leads to a rapid convergence of the subroutine within the range from 1 GHz to 5.5 GHz. Within this latter range, and especially at its center, the calibration constants determined by the exact procedure were near to those determined by the approximate one. Outside this range the deviation starts to increase and in particular on the high-frequency side. Here also, and for the same reasoning, the two calibration groups mentioned earlier were only used for the primary calibration of the reflectometer. The situation is shown in Table II where the results of the exact and approximate procedures are compared at 0.5 GHz, 3 GHz, and 6 GHz. It is clear from this table that the approximate procedure is quite adequate at the center frequency of 3 GHz ($G \doteq 0.0+j0.0$). On the other hand, this procedure becomes less accurate outside the range 1–5.5 GHz, especially on the high-frequency side.

IV. PERFORMANCE OF THE REFLECTOMETER

The performance of the reflectometer is investigated by measuring several terminations by the reflectometer and the HP network analyzers 8505A (0.5–1.3 GHz) and 8410B (2–8 GHz) and comparing the results. The measured terminations consist of a matched load, twelve positions of a sliding short-circuit, and twelve positions of a sliding short-circuit backed by a 3-dB attentuator (both evenly spaced at a 30° interval). The matched load and six positions of the sliding short-circuit were used as the seven calibrating terminations. The reflectometer is then cali brated according to the approximate procedure. The terminations were first measured by the HP network

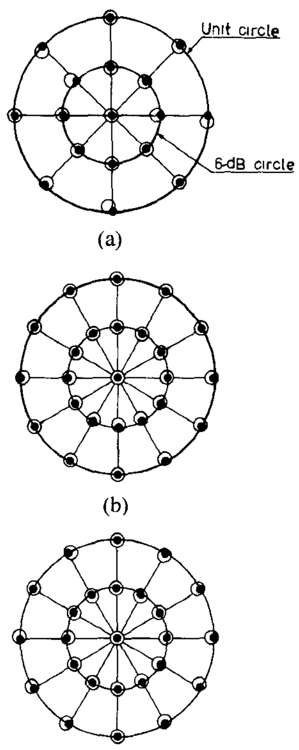

Fig. 5. Comparison of the reflectometer results based on the approximate calibration procedure (∘) and those of the network analyzers (·) at (a) 1.0 GHz, (b) 3.0 GHz, and (c) 5.5 GHz.

analyzers and then by the reflectometer. These measurements were carried out at sixteen frequencies within the frequency range from 0.5 GHz to 8 GHz. The agreement between the reflectometer and network analyzer results is best at 3 GHz. It starts to degrade slightly on both sides of 3 GHz but still generally quite satisfactory within the range from 1 GHz to 5.5 GHz. Outside this range the deviations of the reflectometer results from those of the network analyzers start to increase more rapidly. This is expected due to the deterioration of the behavior of the basic six-port coupler outside this range and especially on the high-frequency side [4]. Three examples of these measurements which were carried out at 1 GHz, 3 GHz, and 5.5 GHz are shown in Fig. 5(a), (b), and (c), respectively. There is an excellent agreement between the reflectometer and the network analyzer results at 3 GHz, and quite satisfactory ones at 1 GHz and 5.5 GHz.

This good agreement confirms the adequacy of the approximate calibration procedure within the range 1–5.5 GHz, as is expected from the results of Section III. As a further check, the reflectometer results of Fig. 5 were reevaluated on basis of the exact calibration, procedure. This reevaluation did not lead to much improvement, especially at 3 GHz. On the other hand, at 0.5 GHz the reflectometer results based on the exact calibration procedure are noticeably better than those based on the approximate one. The situation is shown in Fig. 6. In this figure some reflectometer results deviated from those of the network analyzer. These deviations are most probably due to measurement errors. Here some of the output power levels at ports 3, 4, and 5 are so small that they could be easily masked by noise or errors in the detectors of the power meters (HP 432A). Even the reflectometer results based on the exact calibration procedure (Fig. 6(b)) show similar but lesser deviations. This could be partly due to the same measurement errors and partly due to the results of the adopted minimization subroutine. The input data to this subroutine are the initial guess values obtained from the approximate calibration procedure, and some of them may be far from the true values. This might have caused the subroutine to converge to some false minima, as it is the case with such subroutines, and thus lead to erroneous results. At 6 GHz the reflectometer results, especially those based on the approximate procedure, start to have large deviations from those of the network analyzer. This is due to the increased reflections at the ports 1, 3, 4, and 6 of the basic six-port coupler.

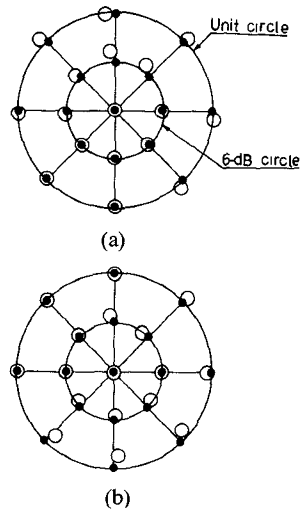

Fig. 6. Comparison of the results of the reflectometer (∘) and the network analyzer (·) at 0.5 GHz by using (a) the approximate and (b) the exact calibration procedure.

V. Useful Bandwidth of the Reflectometer

As mentioned in Section III, the reflection coefficient of the measured impedance Γ_u is determined from the intersection of the corresponding three impedance-locating circles. These circles should, theoretically, intersect in a single point which determine Γ_u both in phase and magnitude. Practically, however, these circles will not, generally, intersect in a single point due to measurement errors, e.g., noise or errors in the detectors of the power meters. Therefore, it is necessary to assign a value to Γ_u in this case. In the present work, this was done in the following way. The two points of intersection of each two of the three circles were found. Only one of each two of these points is a required point that might represent or is near to Γ_u. Thus the required three points should have coordinates that are near to each other. Therefore, it is easy to exclude the other three points. Averaging of the coordinates of the selected three points would be a good estimate for the point that represent Γ_u. This was carried out graphically for the sake of quick illustration of the reflectometer performance. This procedure can also be easily programmed to obtain Γ_u numerically. This is an important point since the reflectometer is to be integrated with a microprocessor. This microprocessor will control the whole process together with carrying out all the necessary computations.

The reflectometer results determined in the above manner were compared with those of the network analyzer at sixteen frequencies within the range 0.5–8 GHz. From this comparison, examples of which are shown in Fig. 5, it was found that a very good accuracy is obtainable in the range 2–4 GHz and a generally quite good one within the range 1–5.5 GHz. This can be extended down to 0.5 GHz by using the exact calibration procedure at this particular frequency. Thus the useful bandwidth of the reflectometer could be from 0.5 GHz to 5.5 GHz.

VI. Conclusions

A six-port reflectometer, using only one well-developed microstrip coupler, was calibrated by a procedure which is based on the approximate equations of the reflectometer. Comparison of the reflectometer results based on this procedure and those based on an "exact procedure" showed the adequacy of the "approximate procedure" within almost the whole useful bandwidth of the reflectometer. Thus a good compromise between the behavior of the reflectometer hardware and the adopted software has proven to be useful.

The positions of the centers of the impedance-locating circles, except that of the first one, change almost linearly with frequency. This feature allows interpolation between the calibration frequencies without significantly affecting accuracy.

The maximum frequency sensitivity of these positions is shown in Table I. For a typical frequency stability of HF generators ($1:10^4$), the corresponding uncertainties in these positions are one order of magnitude less than the values of Table I and are satisfactory.

The reflectometer results are generally in a good agreement with those of the network analyzers within the useful bandwidth of the reflectometer. This bandwidth is found to be from 1 GHz to 5.5 GHz and can be easily extended down to 0.5 GHz.

By frequency scaling, a similar reflectometer could be designed to operate within a higher frequency range making use of the information of this paper and [3] and [4]. No attempt at broadbanding was made at this stage because the main interest was to test the capabilities of the present configuration. However, the usual broadbanding techniques of $\lambda/4$ couplers could be tried.

Appendix

It is obvious from the expressions of (1) that the constants in each of them are different from those of the other two expressions. Thus the constants of each of these expressions can be determined separately in the following manner. The successive connection of four known terminations to port 6 provides four sets of the expressions of (1). Thus we will have four equations for each of the expressions of (1), which are more than sufficient to determine the three real constants of each of them. As an example, let us illustrate the procedure of getting the constants of the first expression in (1) of Section II. The four equations corresponding to this expression are as follows:

$$P_3(1)/P_0 = |A|^2\left((\Gamma_{R1}+x_1)^2+(\Gamma_{I1}+Y_1)^2\right) \quad \text{(A1)}$$

$$P_3(2)/P_0 = |A|^2\left((\Gamma_{R2}+x_1)^2+(\Gamma_{I2}+Y_1)^2\right) \quad \text{(A2)}$$

$$P_3(3)/P_0 = |A|^2\left((\Gamma_{R3}+x_1)^2+(\Gamma_{I3}+Y_1)^2\right) \quad \text{(A3)}$$

$$P_3(4)/P_0 = |A|^2\left((\Gamma_{R4}+x_1)^2+(\Gamma_{I4}+Y_1)^2\right) \quad \text{(A4)}$$

where Γ_R's and Γ_I's are the real and imaginary parts of the reflection coefficients of the used four known terminations, P_3's are the corresponding readings of the power meter at port 3, $-x_1$ and $-Y_1$ are the coordinates of the center of the first impedance-locating circle.

Dividing (A1) by (A2) and rearranging of terms leads to the following circle equation:

$$x_1^2+Y_1^2+2B_1x_1+2C_1Y_1+D_1=0 \quad \text{(A5)}$$

where

$$B_1=(\Gamma_{R2}\cdot A(1)-\Gamma_{R1})/A_1, \quad C_1=(\Gamma_{I2}\cdot A(1)-\Gamma_{I1})/A_1$$
$$D_1=(|\Gamma_2|^2\cdot A(1)-|\Gamma_1|^2)/A_1, \quad A(1)=P_3(1)/P_3(2)$$
$$A_1=A(1)-1.$$

Similarly, (A2) and (A3), (A3) and (A4), (A4) and (A1), (A1) and (A3), and (A2) and (A4) are combined to give, respectively, the following five circle equations:

$$x_1^2+Y_1^2+2B_2x_1+2C_2Y_1+D_2=0 \quad \text{(A6)}$$

$$x_1^2+Y_1^2+2B_3x_1+2C_3Y_1+D_3=0 \quad \text{(A7)}$$

$$x_1^2+Y_1^2+2B_4x_1+2C_4Y_1+D_4=0 \quad \text{(A8)}$$

$$x_1^2+Y_1^2+2B_5x_1+2C_5Y_1+D_5=0 \quad \text{(A9)}$$

$$x_1^2+Y_1^2+2B_6x_1+2C_6Y_1+D_6=0. \quad \text{(A10)}$$

Solving any pair of the above circles should, theoretically, give the required values of x_1 and Y_1. However, the pairs (A5) and (A6), (A6) and (A7), (A7) and (A8), (A8) and (A10), and (A9) and (A10) are solved together resulting in ten points of intersection. These pairs are chosen to avoid repeated solution of two circles based on the same equations among the equations (A1)–(A4). Theoretically, only five of the intersection points should be the same and each should give the required values of x_1 and Y_1. However, due to measurement errors, mentioned in the text, these points may deviate from each other. Therefore, it is necessary to select the point that gives the best solution for x_1 and Y_1. Thus each of the points of intersection are substituted in turn into each of the four equations (A1)–(A4). Then the point that gives the best agreement between the values of $|A|$ obtained from these equations is the required one.

The whole procedure is then repeated with the four known terminations of the second calibration group. Since the matched load is common between the two groups, we will have three extra equations to the previous ones, or a total of seven equations. The values of x_1 and Y_1 obtained from each of the two groups are then substituted in the extra three equations of the other group. The values that still give a good agreement between the values of $|A|$ obtained from the three extra equations of the other group are the best solution for x_1 and Y_1. Sometimes an average of the values of x_1 and Y_1 obtained from the two groups gives this best solution. In this way, the values of $|A|$, x_1 which represents the real part of the complex constant B and Y_1 which represents its imaginary part are determined.

In a similar manner the values of the constants $|C|$, D, $|E|$, and F of the other two expressions of (1) can be determined.

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